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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT 967

ELASTIC AND PLASTIC BUCKLING OF SIMPLY SUPPORTED SOLID-CORE SANDWICH PLATES IN COMPRESSION

By PAUL SEIDE and ELBRIDGE Z. STOWELL

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft (or mi)
Time	t	second	s	second (or hour)	sec (or hr)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb
Power	P	horsepower (metric)		horsepower	hp
Speed	V	kilometers per hour	kph	miles per hour	mph
		meters per second	mps	feet per second	fps

2. GENERAL SYMBOLS

W Weight $=mg$

g Standard acceleration of gravity $=9.80665$ m/s^2 or 32.1740 ft/sec^2

m Mass $=\frac{W}{g}$

I Moment of inertia $=mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ Coefficient of viscosity

ν Kinematic viscosity

ρ Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C and 760 mm; or 0.002378 lb-ft^{-4} sec^2

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb/cu ft

3. AERODYNAMIC SYMBOLS

S Area

S_w Area of wing

G Gap

b Span

c Chord

A Aspect ratio, $\frac{b^2}{S}$

V True air speed

q Dynamic pressure, $\frac{1}{2}\rho V^2$

L Lift, absolute coefficient $C_L=\frac{L}{qS}$

D Drag, absolute coefficient $C_D=\frac{D}{qS}$

D_0 Profile drag, absolute coefficient $C_{D_0}=\frac{D_0}{qS}$

D_i Induced drag, absolute coefficient $C_{D_i}=\frac{D_i}{qS}$

D_p Parasite drag, absolute coefficient $C_{D_p}=\frac{D_p}{qS}$

C Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

i_w Angle of setting of wings (relative to thrust line)

i_t Angle of stabilizer setting (relative to thrust line)

Q Resultant moment

Ω Resultant angular velocity

R Reynolds number, $\rho\frac{Vl}{\mu}$ where l is a linear dimension (e.g., for an airfoil of 1.0 ft chord, 100 mph, standard pressure at 15° C, the corresponding Reynolds number is 935,400; or for an airfoil of 1.0 m chord, 100 mps, the corresponding Reynolds number is 6,865,000)

α Angle of attack

ϵ Angle of downwash

α_0 Angle of attack, infinite aspect ratio

α_i Angle of attack, induced

α_a Angle of attack, absolute (measured from zero-lift position)

γ Flight-path angle

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By PAUL SEIDE and ELBRIDGE Z. STOWELL

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REPORT 967

ELASTIC AND PLASTIC BUCKLING OF SIMPLY SUPPORTED SOLID-CORE SANDWICH PLATES IN COMPRESSION

By Paul Seide and Elbridge Z. Stowell

SUMMARY

A solution is presented for the problem of the compressive buckling of simply supported, flat, rectangular, solid-core sandwich plates stressed either in the elastic range or in the plastic range. Charts for the analysis of long sandwich plates are presented for plates having face materials of 24S–T3 aluminum alloy, 75S–T6 Alclad aluminum alloy, and stainless steel.

A comparison of computed and experimental buckling stresses of square solid-core sandwich plates indicates fair agreement between theory and experiment.

INTRODUCTION

The necessary condition that the wing surfaces of modern high-speed aircraft remain smooth under high loads has led to the use of the sandwich plate as a substitute for sheet-stringer construction. Sandwich plates consist of two thin sheets of metal separated by a low-density, low-stiffness core which, though contributing little to the strength of the plate, serves to increase tremendously the flexural stiffness of the load-carrying faces. The increase in flexural stiffness is somewhat offset, however, by deflections due to shear which become appreciable because of the low stiffness of the core.

Several papers, which extend ordinary plate theory to take into account deflections due to shear, have appeared recently in this country. The extension is made approximately in reference 1 by means of the assumption that any line in the core that is initially straight and normal to the middle surface of the core will remain straight after deformation but will deviate from the normal to the deformed middle surface by an amount that is proportional to the slope of the plate surface, the proportionality factor being the same throughout the plate. The theory is used to obtain approximate criterions for the compressive buckling of plates with various edge-support conditions. The criterions are corrected for the effects of plasticity by replacing the Young's modulus of the face material everywhere it appears in the buckling formulas by a reduced modulus, this method of correction being partly justified by the consideration of its theoretical effectiveness in connection with the plastic buckling of simply supported sandwich columns. Reference 2 presents a small-deflection theory for elastic bending and buckling of orthotropic sandwich plates which considers shear deformations in a more refined manner. Reference 3 presents a large-deflection analysis of elastic isotropic sandwich plates and reduces the equations to small-deflection form to solve the problem of the compressive buckling of simply supported sandwich plates. The theories of references 2 and 3 can be shown to reduce to that of reference 1 in the case of the problem of the compressive buckling of simply supported plates.

In the present report the theory of reference 2 is applied to the problem of the compressive buckling of simply supported solid-core sandwich plates. The particular sandwich considered (fig. 1) is one for which face-parallel stresses in the core may be neglected so that all the applied load is carried by the faces. Furthermore, the faces are assumed to be very thin compared with the core. The stability criterion obtained is similar to those given in references 1 and 3. The theory is also extended to the plastic range in much the same manner as was done in reference 4 for solid plates and is used to determine the plastic compressive buckling stress of simply supported solid-core sandwich plates. Charts for the analysis of long sandwich plates stressed in the elastic range or in the plastic range are presented for plates having face materials of 24S–T3 aluminum alloy, 75S–T6 Alclad aluminum alloy, and stainless steel.

The theory is checked by a comparison of computed and experimental results for square sandwich plates with 24S–T Alclad aluminum-alloy faces and end-grain balsa cores. The experimental results were obtained from reference 5. Fair agreement is found between theory and experiment.

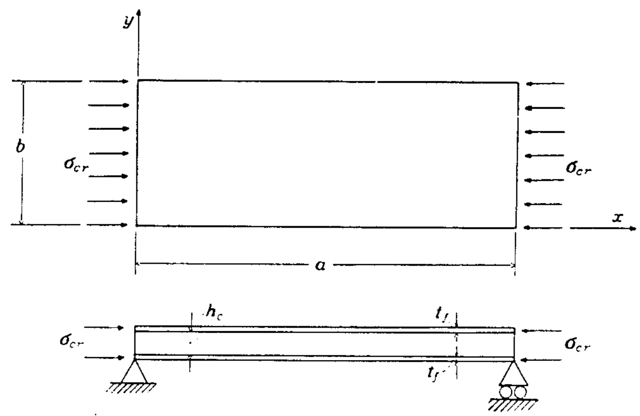


Figure 1.—Simply supported solid-core sandwich plate under compression.

SYMBOLS

x,y	coordinate axes (fig. 1)
E_f	Young's modulus for face material
E_S	secant modulus for face material
E_T	tangent modulus for face material
$C_1=\frac{1}{4}+\frac{3}{4}\frac{E_T}{E_S}$	
$\psi=\frac{E_S}{E_f}$	
μ_f	Poisson's ratio for face material
G_c	shear modulus of core material
t_f	face thickness
h_c	core thickness
D	flexural stiffness per unit width of sandwich plate $\left(\frac{E_f t_f (h_c+t_f)^2}{2(1-\mu_f^2)}\right)$
B	flexural stiffness per unit width of sandwich beam $\left(\frac{E_f t_f (h_c+t_f)^2}{2}\right)$
a	plate length
b	plate width
β	plate aspect ratio (a/b)
σ_{cr}	buckling stress
k	elastic-buckling-stress coefficient $\left(\frac{2b^2\sigma_{cr}t_f}{\pi^2 D}\right)$
k'	elastic-buckling-stress coefficient based upon $\mu_f=\frac{1}{2}$ $\left(\frac{3}{2}\frac{b^2\sigma_{cr}t_f}{\pi^2 B}\right)$
k_{pl}	plastic-buckling-stress coefficient $\left(\frac{2b^2\sigma_{cr}t_f}{\pi^2 D}\right)$
k'_{pl}	plastic-buckling-stress coefficient based upon $\mu_f=\frac{1}{2}$ $\left(\frac{3}{2}\frac{b^2\sigma_{cr}t_f}{\pi^2 B}\right)$
r	core shear-stiffness parameter for sandwich plate $\left(\frac{\pi^2 D}{b^2 G_c h_c}\right)$
s	core shear-stiffness parameter for sandwich column $\left(\frac{\pi^2 B}{b^2 G_c h_c}\right)$
m	number of half-waves in buckled plate deflection surface in direction of loading

RESULTS AND DISCUSSION

Compressive buckling formulas for simply supported flat rectangular solid-core sandwich plates are derived in the appendixes for buckling in either the elastic range or in the plastic range. The equation for compressive buckling in the elastic range is obtained in appendix A by use of the theory developed in reference 2. The theory is modified in appendix B to obtain the equation for compressive buckling in the plastic range.

Elastic range.—For finite plates the buckling-stress coefficient is given by equation (A7) of appendix A as follows:

$$k=\frac{\left(\frac{m}{\beta}+\frac{\beta}{m}\right)^2}{1+r\left(1+\frac{m^2}{\beta^2}\right)} \qquad (1)$$

Consecutive integral values of m are substituted into equation (1) until a minimum value of the buckling coefficient is obtained for given values of β and r. For infinite plates the coefficient reduces to

$$k=\frac{4}{(1+r)^2} \quad (r\leqq 1) \qquad (2)$$

and

$$k=\frac{1}{r} \quad (r\geqq 1) \qquad (3)$$

When the core shear stiffness is infinite ($r=0$), equations (1) and (2) reduce to the well-known buckling criterions for isotropic plates with deflections due to shear neglected.

Equations (1) to (3) are presented graphically in figures 2 and 3. Figure 2 shows that the effect of finite core shear stiffness is not only to decrease the buckling stress but also to increase the number of half-waves in the buckled plate. If the core shear-stiffness parameter is equal to or less than 1.0, the wave length of buckle becomes infinitely small, in

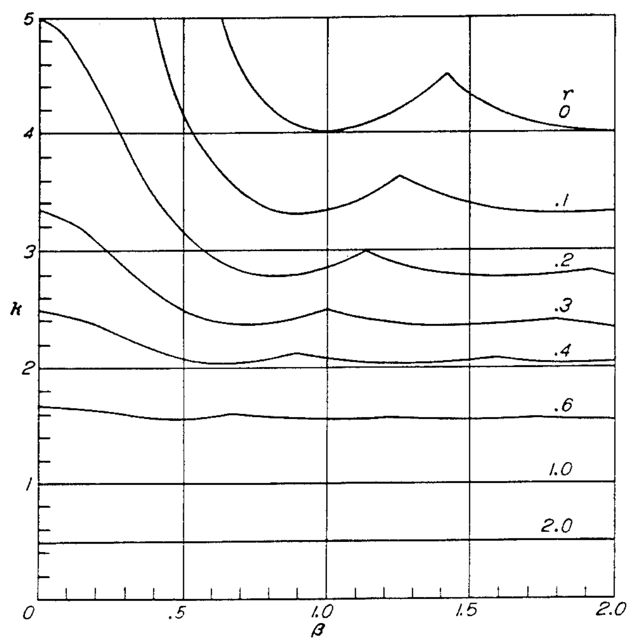


FIGURE 2.—Compressive-buckling coefficients for finite solid-core sandwich plates stressed in the elastic range.

which case the restraint to buckling offered by the side supports has no effect. The buckling-stress coefficient is then independent of the plate aspect ratio β and is determined by the shear strength of the core.

Plastic range.—When the buckling stress is in the plastic range the buckling coefficients are given by the appropriate one of equations (B10) to (B13) of appendix B. Since the buckling coefficient is given by these equations as a function of the buckling stress, a graphical method must be used to analyze a given plate. The buckling coefficient given by equations (B10) to (B13) is defined as

$$k'_{pl}=\frac{3}{2}\frac{b^2\sigma_{cr}t_f}{\pi^2 B} \tag{4}$$

Equation (4) can be rearranged to give

$$\frac{\pi^2 B}{b^2 t_f}=\frac{3}{2}\frac{\sigma_{cr}}{k'_{pl}} \tag{5}$$

so that $\frac{\pi^2 B}{b^2 t_f}$ is now given in terms of the buckling stress, the shear-stiffness parameter $\frac{\pi^2 B}{b^2 G_c h_c}$, and the plate aspect ratio β, all of which are contained in k'_{pl}. For a given value of β, curves of $\frac{\pi^2 B}{b^2 t_f}$ against buckling stress can be plotted for various values of the shear-stiffness parameter $\frac{\pi^2 B}{b^2 G_c h_c}$. Then for a given plate, $\frac{\pi^2 B}{b^2 t_f}$ and $\frac{\pi^2 B}{b^2 G_c h_c}$ are defined by the plate dimensions and material properties and the buckling stress may then be obtained from the appropriate curve.

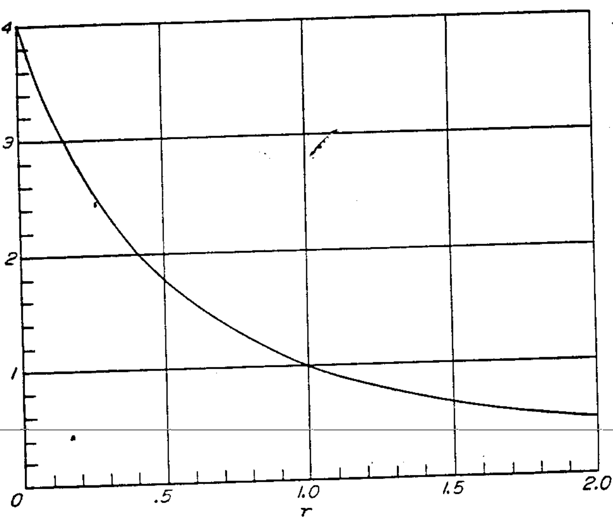


FIGURE 3.—Compressive-buckling coefficients for infinitely long solid-core sandwich plates stressed in the elastic range. $k=\frac{4}{(1+r)^2}$ for $r\leqq 1$; $k=\frac{1}{r}$ for $r\geqq 1$.

Since equations (B10) to (B13) are valid only for plates with a Poisson's ratio of ½, the buckling stresses computed by the foregoing method from those equations are in error for plates having other Poisson's ratios and must be corrected. The correction process used in the present report is the following: For a given plate, the plastic buckling stress based on a Poisson's ratio of ½ is computed by the foregoing method. The buckling stress for a perfectly elastic plate is also computed by using the appropriate one of equations (B14) to (B16) which are also based upon a Poisson's ratio of ½. It is assumed that for given values of $\frac{\pi^2 B}{b^2 t_f}$ and $\frac{\pi^2 B}{b^2 G_c h_c}$ the ratio of the plastic and elastic stresses is independent of Poisson's ratio. Then for any other value of Poisson's ratio the corrected buckling stress is given by:

$$\sigma_{cr}=\eta\times\text{Elastic buckling stress for actual value of }\mu_f$$

$$=\eta\frac{\pi^2 B}{2b^2 t_f}\frac{k}{1-\mu_f^2} \tag{6}$$

where η is the ratio of the plastic and elastic buckling stresses computed on the basis of $\mu_f=\frac{1}{2}$ and k is determined from equations (1) to (3) for finite plates as

$$k=\frac{\left(\frac{m}{\beta}+\frac{\beta}{m}\right)^2}{1+\frac{\frac{\pi^2 B}{b^2 G_c h_c}}{1-\mu_f^2}\left(1+\frac{m^2}{\beta^2}\right)} \tag{7}$$

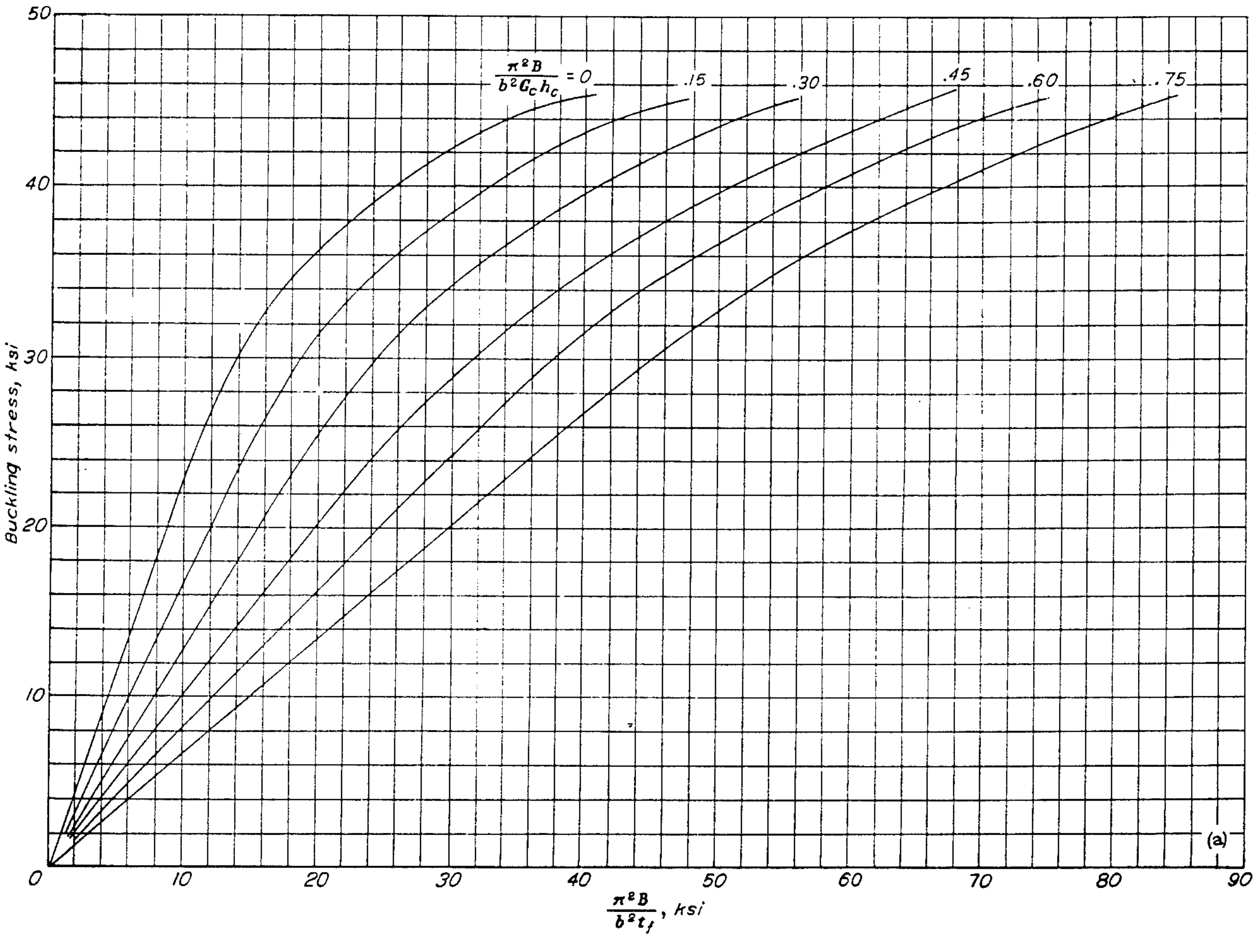
and for infinitely long plates as

$$k=\frac{4}{\left(1+\frac{\frac{\pi^2 B}{b^2 G_c h_c}}{1-\mu_f^2}\right)^2}\qquad\left(\frac{\pi^2 B}{b^2 G_c h_c}\leqq 1-\mu_f^2\right) \tag{8}$$

$$k=\frac{1-\mu_f^2}{\left(\frac{\pi^2 B}{b^2 G_c h_c}\right)}\qquad\left(\frac{\pi^2 B}{b^2 G_c h_c}\geqq 1-\mu_f^2\right) \tag{9}$$

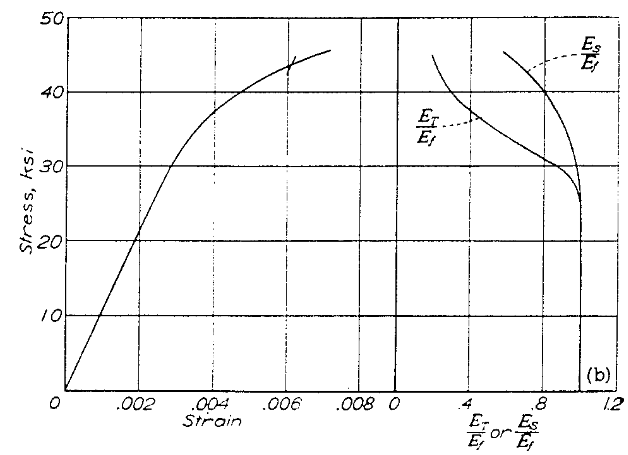
Curves of $\frac{\pi^2 B}{b^2 t_f}$ against the corrected buckling stress for various values of $\frac{\pi^2 B}{b^2 G_c h_c}$ may now be drawn. Different sets of curves are obtained for different values of μ_f.

Charts for the analysis of infinitely long sandwich plates were constructed by the foregoing method for face materials of 24S–T3 aluminum alloy, Alclad 75S–T6 aluminum alloy, and stainless steel and are presented, together with the typical



(a) Design chart.

FIGURE 4.—Charts for long solid-core sandwich plates with 24S-T3 aluminum-alloy faces. $\mu_f=\frac{1}{3}$.



(b) Typical stress-strain relations for 24S-T3 aluminum alloy.

FIGURE 4.—Concluded.

stress-strain curves on which they are based, as figures 4, 5, and 6, respectively. In each case μ_f was taken as ⅓. Since the equations used do not depend on the stress-strain curve itself but upon its shape as given by the curves of E_S/E_f and E_T/E_f as functions of stress (figs. 4(b), 5(b), and 6(b)), solid-core sandwich plates having faces of any material for which curves of E_S/E_f and E_T/E_f against stress are similar to those used may be analyzed by means of the corresponding chart.

The charts of figures 4, 5, and 6 for infinitely long sandwich plates may be used with little error for finite plates, the aspect ratios of which are greater than 3. An extension of the curves of figure 2 would indicate that in this range of aspect ratio the buckling coefficient is essentially given by that for the infinitely long plate, especially if the core shear stiffness is low.

Comparison of theory and experiment.—An experimental check of the equations derived in this report for the compressive buckling of simply supported solid-core sandwich plates was obtained by a comparison of computed and experimental buckling stresses of square plates having 24S-T Alclad aluminum-alloy faces and end-grain-balsa cores of various thicknesses (fig. 7). The experimental results were obtained from reference 5.

The computations involved in the determination of the theoretical stresses were shortened by using the typical stress-strain curve of figure 4(b) for 24S-T3 aluminum alloy instead of the stress-strain curves presented in reference 5 for 24S-T Alclad aluminum-alloy sheet of various thicknesses. The stress-strain curve used is approximately the average of the actual stress-strain curves.

As indicated by figure 7 the agreement between computed and experimental stresses is fair, the computed stresses being on the average 8 percent higher than the experimental stresses. In individual cases, however, the deviation is as

APPENDIX A

DERIVATION OF COMPRESSIVE BUCKLING EQUATION FOR SIMPLY SUPPORTED SOLID-CORE SANDWICH PLATES STRESSED IN THE ELASTIC RANGE

The compressive buckling criterion for simply supported solid-core sandwich plates (fig. 1) stressed in the elastic range may be derived by means of equations (5a) to (6c) of reference 2. In the equations seven physical constants of sandwich plates (two Poisson's ratios, two flexural stiffnesses, a twisting stiffness, and two shear stiffnesses) must be specified. In order to determine the physical constants, the following assumptions are made in the present report:

1. The faces and core are isotropic.
2. Face-parallel stresses in the core may be neglected so that the applied loads are carried only by the faces.
3. Vertical shear forces are carried only by the core and are distributed uniformly across the thickness of the core.
4. The faces are assumed to be very thin compared to the core so that the variation of face-parallel stresses across the thickness of the faces may be neglected.

Under these assumptions the physical constants of solid-core sandwich plates are

$$\left.\begin{aligned} &\mu_x=\mu_y=\mu_f \\ &D_x=D_y=(1+\mu_f)D_{xy}=\frac{E_f t_f(h_c+t_f)^2}{2} \\ &D_{Q_x}=D_{Q_y}=G_c h_c \end{aligned}\right\} \qquad \text{(A1)}$$

Equations (5a) to (6c) of reference 2 may then be written as

$$M_x=-D\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}-\frac{Q_x}{G_c h_c}\right)+\mu_f\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}-\frac{Q_y}{G_c h_c}\right)\right] \qquad \text{(A2a)}$$

$$M_y=-D\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}-\frac{Q_y}{G_c h_c}\right)+\mu_f\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}-\frac{Q_x}{G_c h_c}\right)\right] \qquad \text{(A2b)}$$

$$M_{xy}=\frac{1-\mu_f}{2}D\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}-\frac{Q_y}{G_c h_c}\right)+\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}-\frac{Q_x}{G_c h_c}\right)\right] \qquad \text{(A2c)}$$

$$\frac{\partial^2 M_x}{\partial x^2}-2\frac{\partial^2 M_{xy}}{\partial x\,\partial y}+\frac{\partial^2 M_y}{\partial y^2}=2\sigma_{cr}t_f\frac{\partial^2 w}{\partial x^2} \qquad \text{(A2d)}$$

$$Q_x=-\frac{\partial M_{xy}}{\partial y}+\frac{\partial M_x}{\partial x} \qquad \text{(A2e)}$$

$$Q_y=-\frac{\partial M_{xy}}{\partial y}+\frac{\partial M_y}{\partial y} \qquad \text{(A2f)}$$

where M_x, M_y, M_{xy} are the bending and twisting moments, Q_x, Q_y are the shear forces, and w is the middle-surface deflection at the point (x,y) in the sandwich plate. Equations (A2) constitute the six fundamental differential equations for elastic buckling of solid-core sandwich plates.

An equation in terms of the middle-surface deflection w alone can be obtained. Substitution of the expressions for M_x, M_y, and M_{xy} given in equations (A2a) to (A2c) into equation (A2d) yields

$$-D\nabla^4 w+\frac{D}{G_c h_c}\nabla^2\left(\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}\right)-2\sigma_{cr}t_f\frac{\partial^2 w}{\partial x^2}=0 \qquad \text{(A3)}$$

But, from equations (A2d) to (A2f),

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}=2\sigma_{cr}t_f\frac{\partial^2 w}{\partial x^2} \qquad \text{(A4)}$$

Hence equation (A3) reduces to

$$\nabla^4 w+\left(1-\frac{D}{G_c h_c}\nabla^2\right)\frac{2\sigma_{cr}t_f}{D}\frac{\partial^2 w}{\partial x^2}=0 \qquad \text{(A5)}$$

Equation (A5) is identical with equation (71) of reference 3 for a plate under compression in one direction.

Since the plate is simply supported on all edges, the deflection surface may be taken as

$$w=A\sin\frac{m\pi x}{a}\sin\frac{\pi y}{b} \qquad \text{(A6)}$$

yielding the stability criterion

$$k=\frac{\left(\frac{m}{\beta}+\frac{\beta}{m}\right)^2}{1+r\left(1+\frac{m^2}{\beta^2}\right)} \qquad \text{(A7)}$$

The value of m to be used in equation (A7) is that which yields the lowest value of k for given values of r and β.

Equations for elastic buckling of infinitely long simply supported sandwich plates under compression are obtained by minimizing equation (A7) with respect to β/m. This procedure yields

$$\left.\begin{aligned} &\frac{\beta}{m}=\sqrt{\frac{1-r}{1+r}} \\ &k=\frac{4}{(1+r)^2} \end{aligned}\right\} \qquad (r\leq 1) \qquad \text{(A8)}$$

and

$$\left.\begin{aligned} &\frac{\beta}{m}=0 \\ &k=\frac{1}{r} \end{aligned}\right\} \qquad (r\geq 1) \qquad \text{(A9)}$$

The buckling coefficient given by equation (A9) corresponds to failure of the core material under the action of the core shear forces.

Equations (A7) to (A9) are similar to equations (76), (79), and (79a) of reference 3.

APPENDIX B

DERIVATION OF COMPRESSIVE BUCKLING EQUATION FOR SIMPLY SUPPORTED SOLID-CORE SANDWICH PLATES STRESSED IN THE PLASTIC RANGE

When the faces of sandwich plates are stressed in the plastic range, the buckling theory used in appendix A is no longer applicable. The equations of equilibrium, equations (A2d) to (A2f), remain unchanged but the deformation equations, (A2a) to (A2c), must be modified to include plastic effects. This modification may be readily made by means of the plastic buckling theory of reference 4 which is based on the plastic stress-strain relations characteristic of the deformation theory of plasticity. The stress-strain relations involve the assumptions that the plate material is isotropic and incompressible and that no part of the plate unloads during buckling.

Since in the sandwich plates considered in this report the applied forces are assumed to be carried only by the faces and the stresses arising from these forces are assumed to be distributed uniformly across the thickness of the faces, the bending and twisting moments are given by the equations

$$\left.\begin{aligned} M_x &= (\delta\sigma_x{}^U - \delta\sigma_x{}^L)\frac{t_f(h_c+t_f)}{2} \\ M_y &= (\delta\sigma_y{}^U - \delta\sigma_y{}^L)\frac{t_f(h_c+t_f)}{2} \\ M_{xy} &= -(\delta\tau_{xy}{}^U - \delta\tau_{xy}{}^L)\frac{t_f(h_c+t_f)}{2} \end{aligned}\right\} \qquad \text{(B1)}$$

where $\delta\sigma_x$, $\delta\sigma_y$, $\delta\tau_{xy}$ are small variations of the average stresses in the faces when buckling occurs from their values before buckling. The superscripts U and L refer to the upper and lower faces, respectively. The positive direction of M_{xy} is taken in accordance with that given in reference 1 and is the negative of that given in reference 2.

Expressions for the variations of the average stresses in the faces may be obtained from the general treatment of reference 2. For the case of a plate compressed in the x-direction, these equations are

$$\left.\begin{aligned} \delta\sigma_x &= \frac{4}{3}E_S\left\{\left[\epsilon_1+\frac{1}{2}\epsilon_2-\frac{3}{4}\left(1-\frac{E_T}{E_S}\right)\chi_1 z_0\right]\mp \left(\frac{h_c+t_f}{2}\right)\left(C_1\chi_1+\frac{1}{2}\chi_2\right)\right\} \\ \delta\sigma_y &= \frac{4}{3}E_S\left[\left(\epsilon_2+\frac{1}{2}\epsilon_1\right)\mp\left(\frac{h_c+t_f}{2}\right)\left(\chi_2+\frac{1}{2}\chi_1\right)\right] \\ \delta\tau_{xy} &= \frac{2}{3}E_S\left[\epsilon_3\mp\left(\frac{h_c+t_f}{2}\right)\chi_3\right] \end{aligned}\right\} \qquad \text{(B2)}$$

where

ϵ_1, ϵ_2, ϵ_3 variations of middle-surface strains

χ_1, χ_2, χ_3 parts of plate bending and twisting curvatures that cause stresses in the faces

z_0 coordinate of neutral surface of plate

The upper and lower signs refer to the upper and lower faces of the plate, respectively.

The deformations due to vertical shear consist merely of a sliding of the plate cross sections with respect to one another and hence do not contribute to the face stresses. The curvatures due to shear deflections therefore must be subtracted from the total plate curvatures to give the curvatures used in equations (B2). Then, if the core is assumed to be stressed in the elastic range,

$$\left.\begin{aligned} \chi_1 &= \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}-\frac{Q_x}{G_c h_c}\right) \\ \chi_2 &= \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}-\frac{Q_y}{G_c h_c}\right) \\ \chi_3 &= \frac{1}{2}\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}-\frac{Q_x}{G_c h_c}\right)+\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}-\frac{Q_y}{G_c h_c}\right)\right] \end{aligned}\right\} \qquad \text{(B3)}$$

The substitution of equations (B2) and (B3) into equations (B1) yields the modified deformation equations

$$\left.\begin{aligned} M_x &= -\frac{4}{3}\psi B\left[C_1\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}-\frac{Q_x}{G_c h_c}\right)+\frac{1}{2}\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}-\frac{Q_y}{G_c h_c}\right)\right] \\ M_y &= -\frac{4}{3}\psi B\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}-\frac{Q_y}{G_c h_c}\right)+\frac{1}{2}\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}-\frac{Q_x}{G_c h_c}\right)\right] \\ M_{xy} &= \frac{1}{3}\psi B\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}-\frac{Q_x}{G_c h_c}\right)+\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}-\frac{Q_y}{G_c h_c}\right)\right] \end{aligned}\right\} \qquad \text{(B4)}$$

Equations (B4) together with equations (A2d) to (A2f) of appendix A constitute the six fundamental differential equations for plastic compressive buckling of solid-core sandwich plates. Equations (A2d) to (A2f) are

$$\left.\begin{aligned} \frac{\partial^2 M_x}{\partial x^2}-2\frac{\partial^2 M_{xy}}{\partial x\,\partial y}+\frac{\partial^2 M_y}{\partial y^2} &= 2\sigma_{cr}t_f\frac{\partial^2 w}{\partial x^2} \\ Q_x &= -\frac{\partial M_{xy}}{\partial y}+\frac{\partial M_x}{\partial x} \\ Q_y &= -\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y} \end{aligned}\right\} \qquad \text{(B5)}$$

Unlike the elastic buckling theory, the theory for plastic buckling does not yield a single equation in the middle-surface deflection w. The number of equations necessary for the determination of the compressive buckling load may be reduced to three if equations (B4) are substituted into equations (B5), so that

$$\left.\begin{aligned}
&\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}-2\sigma_{cr}t_f\frac{\partial^2 w}{\partial x^2}=0\\
&\left(\frac{1}{4}\frac{\partial^2}{\partial y^2}+C_1\frac{\partial^2}{\partial x^2}-\frac{3}{4}\frac{G_ch_c}{\psi B}\right)\frac{Q_x}{G_ch_c}+\frac{3}{4}\frac{\partial^2}{\partial x\,\partial y}\frac{Q_y}{G_ch_c}-\\
&\frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial y^2}+C_1\frac{\partial^2}{\partial x^2}\right)w=0\\
&\frac{3}{4}\frac{\partial^2}{\partial x\,\partial y}\frac{Q_x}{G_ch_c}+\left(\frac{1}{4}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-\frac{3}{4}\frac{G_ch_c}{\psi B}\right)\frac{Q_y}{G_ch_c}-\\
&\frac{\partial}{\partial y}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w=0
\end{aligned}\right\}\qquad\text{(B6)}$$

The conditions that must be satisfied at the edges of a simply supported sandwich plate are

$$\left.\begin{aligned}
&w=M_x=\frac{Q_y}{G_ch_c}=0 \qquad (\text{at } x=0, a)\\
&w=M_y=\frac{Q_x}{G_ch_c}=0 \qquad (\text{at } y=0, b)
\end{aligned}\right\}\qquad\text{(B7)}$$

Solutions of equations (B6) that satisfy these boundary conditions are

$$\left.\begin{aligned}
&w=A_1\sin\frac{m\pi x}{a}\sin\frac{\pi y}{b}\\
&\frac{Q_x}{G_ch_c}=A_2\cos\frac{m\pi x}{a}\sin\frac{\pi y}{b}\\
&\frac{Q_y}{G_ch_c}=A_3\sin\frac{m\pi x}{a}\cos\frac{\pi y}{b}
\end{aligned}\right\}\qquad\text{(B8)}$$

Substitution of equations (B8) in equations (B6) yields the set of equations

$$\left.\begin{aligned}
&\frac{m\pi}{a}\left[\left(\frac{\pi}{b}\right)^2+C_1\left(\frac{m\pi}{a}\right)^2\right]A_1-\left[\frac{1}{4}\left(\frac{\pi}{b}\right)^2+C_1\left(\frac{m\pi}{a}\right)^2+\frac{3}{4}\frac{G_ch_c}{\psi B}\right]A_2-\frac{3}{4}\frac{m\pi}{a}\frac{\pi}{b}A_3=0\\
&\frac{\pi}{b}\left[\left(\frac{\pi}{b}\right)^2+\left(\frac{m\pi}{a}\right)^2\right]A_1-\frac{3}{4}\frac{m\pi}{a}\frac{\pi}{b}A_2-\left[\frac{1}{4}\left(\frac{m\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2+\frac{3}{4}\frac{G_ch_c}{\psi B}\right]A_3=0\\
&\frac{2\sigma_{cr}t_f}{B}\frac{B}{G_ch_c}\left(\frac{m\pi}{a}\right)^2A_1+\frac{m\pi}{a}A_2+\frac{\pi}{b}A_3=0
\end{aligned}\right\}\qquad\text{(B9)}$$

Since A_1, A_2, and A_3 must have values other than zero, setting the determinant of the coefficients of A_1, A_2, and A_3 equal to zero yields the stability criterion

$$k'_{pl}=\psi\frac{\left(\frac{\beta}{m}\right)^6\left(1+\frac{1}{3}\psi s\right)+\left(\frac{\beta}{m}\right)^4\left[2+\frac{1}{3}\psi s\,(4C_1-1)\right]+\left(\frac{\beta}{m}\right)^2\left[C_1+\frac{1}{3}\psi s\,(5C_1-2)\right]+\frac{1}{3}C_1\psi s}{\left(\frac{\beta}{m}\right)^4\left(1+\frac{5}{3}\psi s+\frac{4}{9}\psi^2s^2\right)+\left(\frac{\beta}{m}\right)^2\left[\frac{1}{3}\psi s\,(4C_1+1)+\frac{8}{9}\psi^2s^2\,(2C_1-1)\right]+\frac{4}{9}C_1\psi^2s^2}\qquad\text{(B10)}$$

The plastic compressive buckling load of infinitely long sandwich plates may be obtained by minimizing equation (B10) with respect to β/m. This procedure yields

$$\left(\frac{\beta}{m}\right)^8\left[\left(1+\frac{1}{3}\psi s\right)^2\left(1+\frac{4}{3}\psi s\right)\right]+\left(\frac{\beta}{m}\right)^6\left\{\frac{2}{3}\psi s\left(1+\frac{1}{3}\psi s\right)\left[1+4C_1+\frac{8}{3}\psi s\left(2C_1-1\right)\right]\right\}+\left(\frac{\beta}{m}\right)^4\left[\frac{8}{27}\psi^3s^3\left(8C_1^2-7C_1+2\right)+\frac{1}{9}\psi^2s^2\left(16C_1^2+15C_1-7\right)+\frac{2}{3}\psi s\left(2-C_1\right)-C_1\right]+\left(\frac{\beta}{m}\right)^2\left\{\frac{2}{3}C_1\psi s\left[\frac{8}{9}\psi^2s^2\left(2C_1-1\right)+\psi s-1\right]\right\}+\frac{1}{9}C_1\psi^2s^2\left(\frac{4}{3}C_1\psi s-1\right)=0\qquad\text{(B11)}$$

Equations (B10) and (B11) then determine the compressive buckling load of infinitely long sandwich plates. For any given values of the buckling stress and the shear stiffness parameter s, equation (B11) is used to find the value of β/m that yields the minimum value of k'_{pl}. This value of β/m is then substituted in equation (B10) to determine k'_{pl}. If all the values of β/m given by equation (B11) are imaginary; that is, if

$$s>\frac{3}{4}\frac{1}{C_1\psi}\qquad\text{(B12)}$$

β/m must be taken equal to zero in equation (B10), which becomes

$$k'_{pl}=\frac{1}{4s/3} \tag{B13}$$

which is identical with equation (A9) if Poisson's ratio is taken equal to ½ in equation (A9).

If the buckling stress is in the elastic range, C_1 and ψ are equal to unity and the equations for compressive buckling of finite solid-core sandwich plates reduce to

$$k'=\frac{\left(\frac{m}{\beta}+\frac{\beta}{m}\right)^2}{1+\frac{4}{3}s\left(1+\frac{m^2}{\beta^2}\right)} \tag{B14}$$

and for compressive buckling of infinitely long sandwich plates,

$$\left.\begin{aligned} k'&=\frac{4}{\left(1+\frac{4}{3}s\right)^2} \\ \frac{\beta}{m}&=\sqrt{\frac{1-\frac{4}{3}s}{1+\frac{4}{3}s}} \end{aligned}\right\} \quad \left(s\leqq\frac{3}{4}\right) \tag{B15}$$

and

$$\left.\begin{aligned} k'&=\frac{1}{4s/3} \\ \frac{\beta}{m}&=0 \end{aligned}\right\} \quad \left(s\geqq\frac{3}{4}\right) \tag{B16}$$

Equations (B14) to (B16) are identical with equations (A7) to (A9) if Poisson's ratio in equations (A7) to (A9) is taken to be ½.

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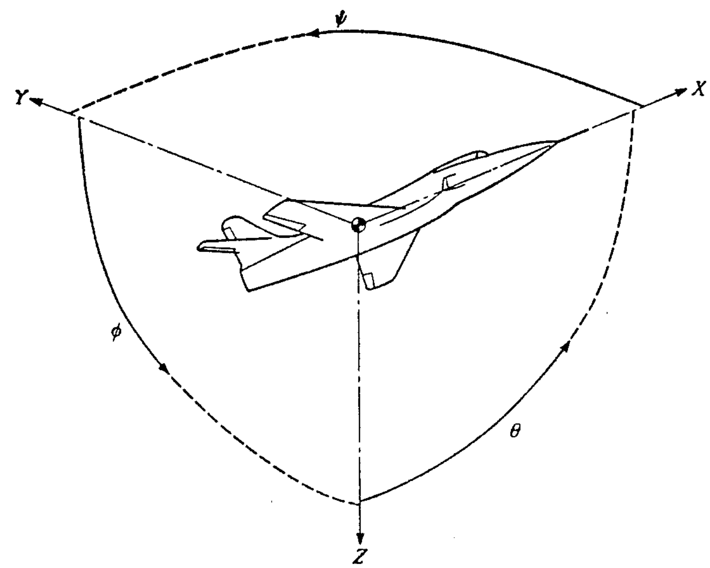

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l=\frac{L}{qbS}$ (rolling) $C_m=\frac{M}{qcS}$ (pitching) $C_n=\frac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D Diameter
p Geometric pitch
p/D Pitch ratio
V' Inflow velocity
V_s Slipstream velocity
T Thrust, absolute coefficient $C_T=\frac{T}{\rho n^2 D^4}$
Q Torque, absolute coefficient $C_Q=\frac{Q}{\rho n^2 D^5}$

P Power, absolute coefficient $C_P=\frac{P}{\rho n^3 D^5}$
C_s Speed-power coefficient $=\sqrt[5]{\frac{\rho V^5}{P n^2}}$
η Efficiency
n Revolutions per second, rps
Φ Effective helix angle $=\tan^{-1}\left(\frac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS

1 hp=76.04 kg-m/s=550 ft-lb/sec
1 metric horsepower=0.9863 hp
1 mph=0.4470 mps
1 mps=2.2369 mph

1 lb=0.4536 kg
1 kg=2.2046 lb
1 mi=1,609.35 m=5,280 ft
1 m=3.2808 ft